Figures of Merit for Light Bucket Mirrors

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Topics

- Background and motivation
- Circle of Confusion
- Aberration characterizations
- Figures of Merit
- Present conclusions

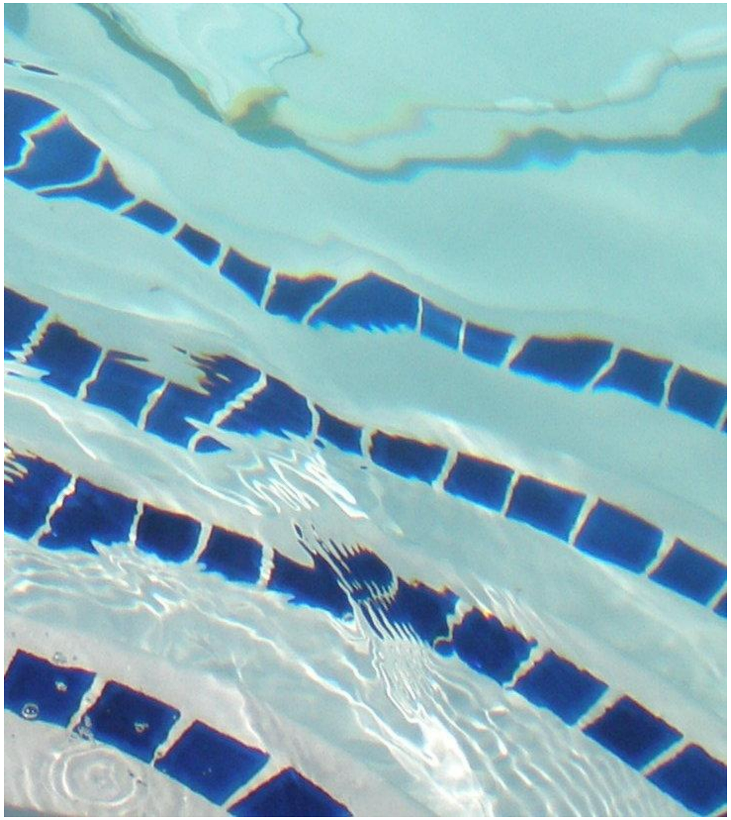
Background & Motivation I

- Pneumatic mirrors for astronomy
  - Study started in 1991 at the U. of Pennsylvania and continued there through 1998
  - Resurrected at Gravic in 2008 for ground-based light buckets
  - Science interests – Intensity interferometry, occultations, high speed aperture photometry

Gravic 42” on IPI393 GEM

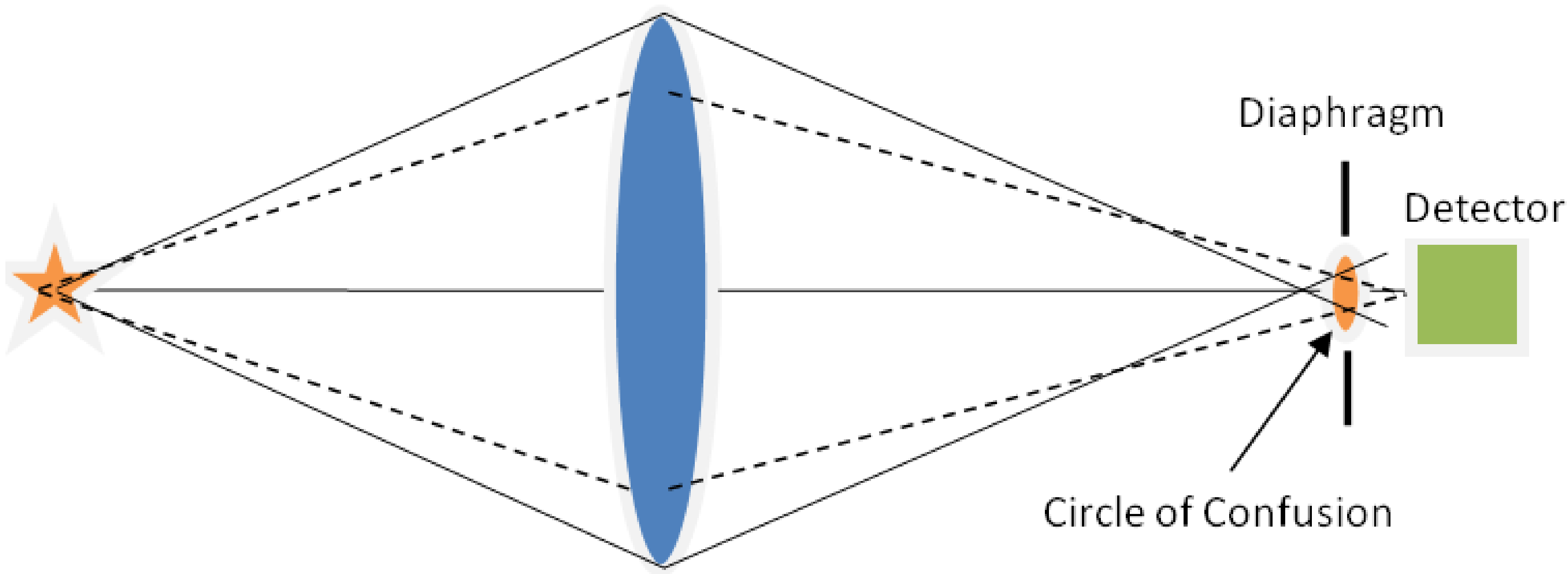
Background & Motivation II

- Tools were needed to characterize progress and failure in our work
  - Traditional quantification such as P-V and Strehl Ratio were not helpful
  - "Highly aberrated" to us signifies many waves of caustic, ray-crossing aberrations



Pool caustics

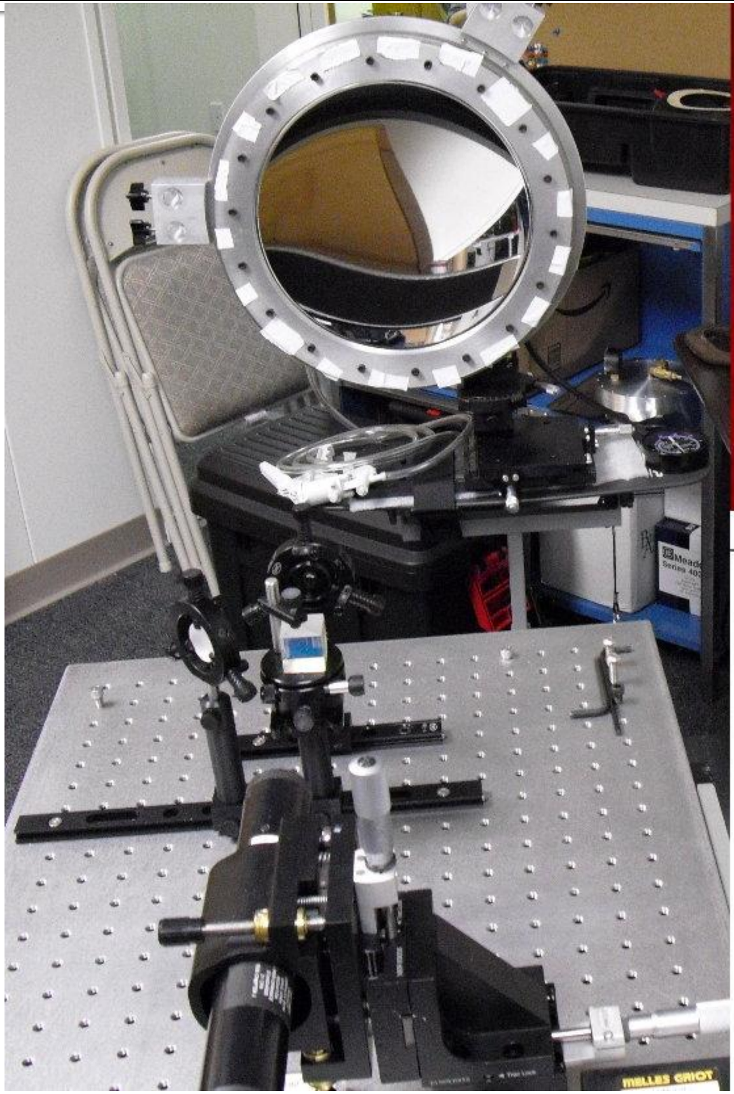
Circle of Confusion

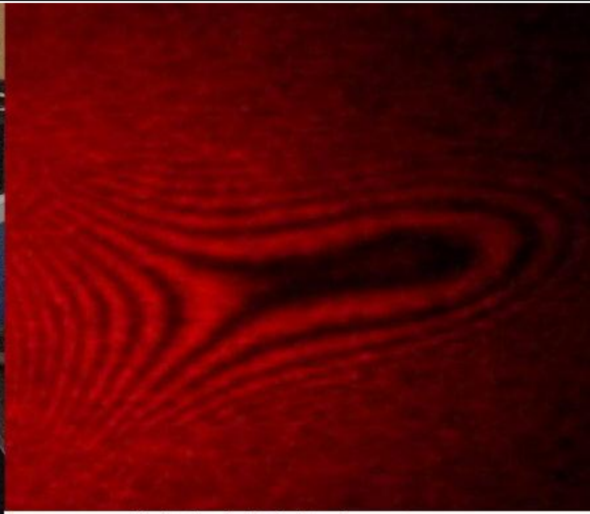


- Circle of Confusion = blur spot at focal plane
- Diaphragm = circular isolator before the detector

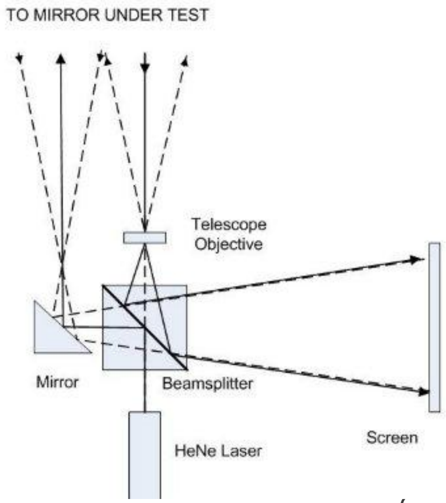
Aberration Characterization I

- Zone-sampling with a Right-angle Bath Interferometer
- Analysis produces Zernike representation of wavefront , $W(\rho,\theta)$
- Statistical combination of sample zone results



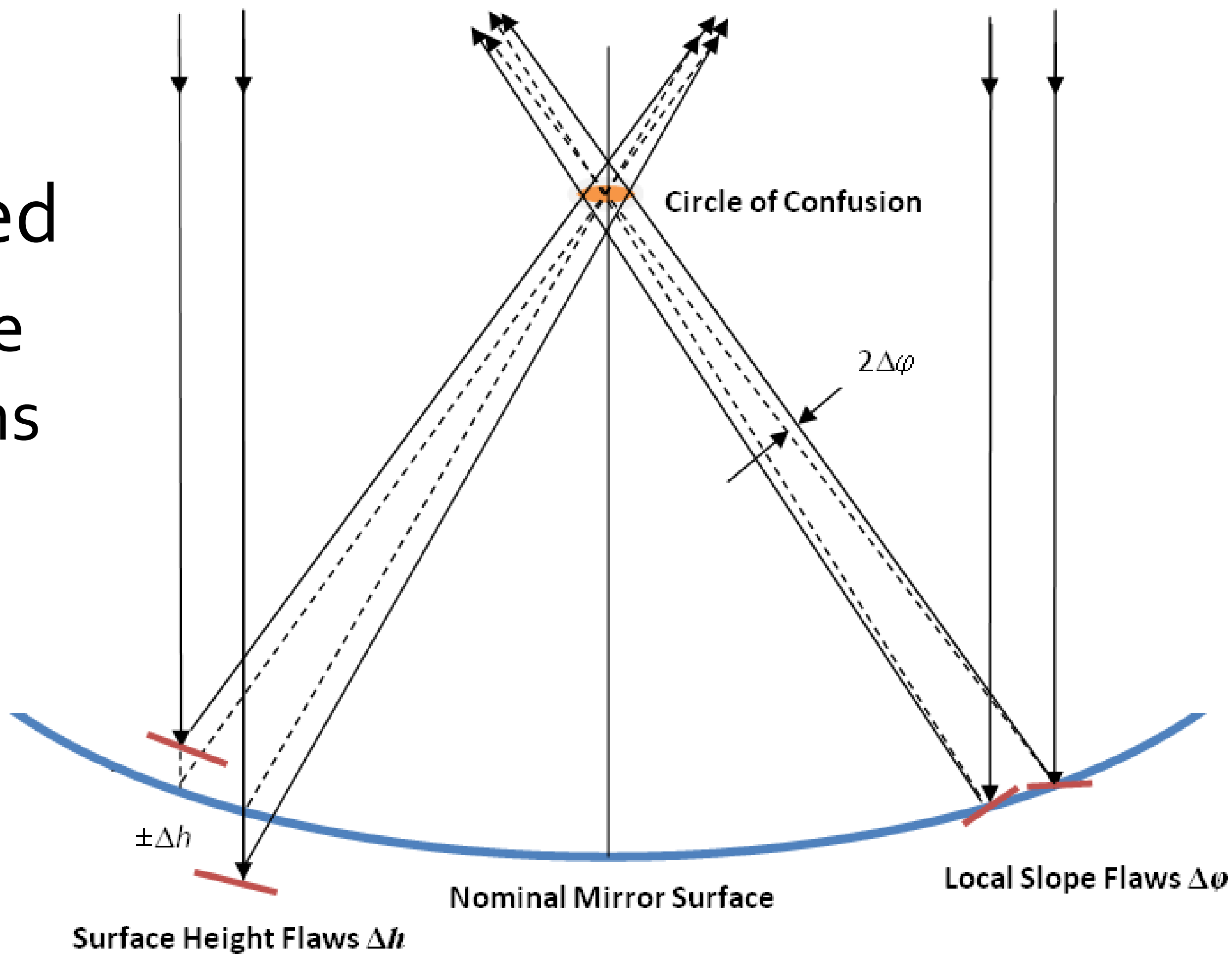


Right-Angle Bath Interferometer



Aberration Characterization II

- Two aberration types considered
  - Random surface height variations
  - Random local slope problems



Aberration Characterization III

- Diameter of CoC from surface height flaws:

$$d_{CoC,surface\ height}(n) \approx\ 0.5n\sigma/f$$

- Diameter of CoC from local slope flaws:

$$d_{CoC,local\ slope}(n') \approx 4n'F\ |\Delta\phi|_{rms}$$

where *f* is the focal ratio, *F* is the focal length, and the *n and n′* multipliers determine the encircled flux fraction

Aberration Characterization IV

- Zernike wavefront representation, *W(ρ,θ)*, is used for the estimation of **σ** and $|\boldsymbol{\Delta\varphi}|_{\mathbf{rms}}$
- **1** $$W(\rho,\theta) = \sum_j a_j Z_j(\rho,\theta)$$
- **2** $$\sigma_W^2 = \langle W^2(\rho,\theta)\rangle - \langle W(\rho,\theta)\rangle^2 = \sum_{i=2} a_j^2$$
- **3** $$\boldsymbol{\nabla} W(\rho,\theta) = \frac{\delta W}{\delta\rho}\,\boldsymbol{e_\rho} + \frac{1}{\rho}\frac{\delta W}{\delta\theta}\,\boldsymbol{e_\theta}$$
- **4** $$|\Delta\phi|_{rms} = \frac{\|\boldsymbol{\nabla} W\|_{rms}}{D/2}$$

Aberration Characterization V

- Calculation of the rms wavefront gradient norm from Zernike coefficients (Southwell 1982, Braat 1987)

$$\langle \|\nabla W\|^2 \rangle = \sum_{l=1}^{\infty} 8l \left[\sum_{i=l}^{\infty} \sqrt{2i+1}\, a_{2i}^{0} \right]^2 + \\ + \sum_{m=1}^{\infty} \left\{ m \left[\sum_{i=0}^{\infty} \sqrt{2(2i+m+1)}\, a_{2i+m}^{m} \right]^2 + \right. \\ \left. + \sum_{l=1}^{\infty} 2(2l+m) \left[\sum_{i=l}^{\infty} \sqrt{2(2i+m+1)}\, a_{2i+m}^{m} \right]^2 \right\}$$

- *FringeXP* (Rowe 2003) coefficient form

$$\|\nabla W\|_{rms} \approx \left[Z_1^2 + 2Z_1Z_6 + Z_2^2 + 2Z_2Z_7 + 8Z_3^2 + 16Z_3Z_8 + 2Z_4^2 + 2Z_5^2 + 7Z_6^2 + 7Z_7^2 + \right. \\ \left. + 24Z_8^2 + 3Z_9^2 + 3Z_{10}^2 \right]^{\frac{1}{2}} .$$

Southwell, W. H. 1982, *Proc. SPIE*, **365**, pp. 97-104

Braat, J. 1987, *J. Opt. Soc. Am.*, **A4**, pp. 643-650

Aberration Characterization VI

- How much aberration is permissible?

For surface height flaws, the **rms wavefront error** must not exceed

$$\sigma_{\text{limit}} \approx 2 f d_{Diaphragm}/n$$

An *f*/2 mirror with 1.3-mm rms smooth surface height aberrations (*i.e.*, 2600 waves of 500-nm light) feeding a 1-mm diameter diaphragm encircles 99.7% of the flux (*n*=3).

Aberration Characterization VII

For local slope flaws, the **rms wavefront gradient norm** must not exceed

$$\|\nabla W\|_{rms,limit} \approx \frac{d_{Diaphragm}}{8n'f}$$

An *f*/2 mirror with a 1-mm diaphragm tolerates 42-waves (500-nm) rms wavefront gradient norm aberration and still encircles 98.9% of the flux (*n*′=3).

Figures of Merit I

- How do aberrations affect the Signal-to-Noise-Ratio (SNR)?

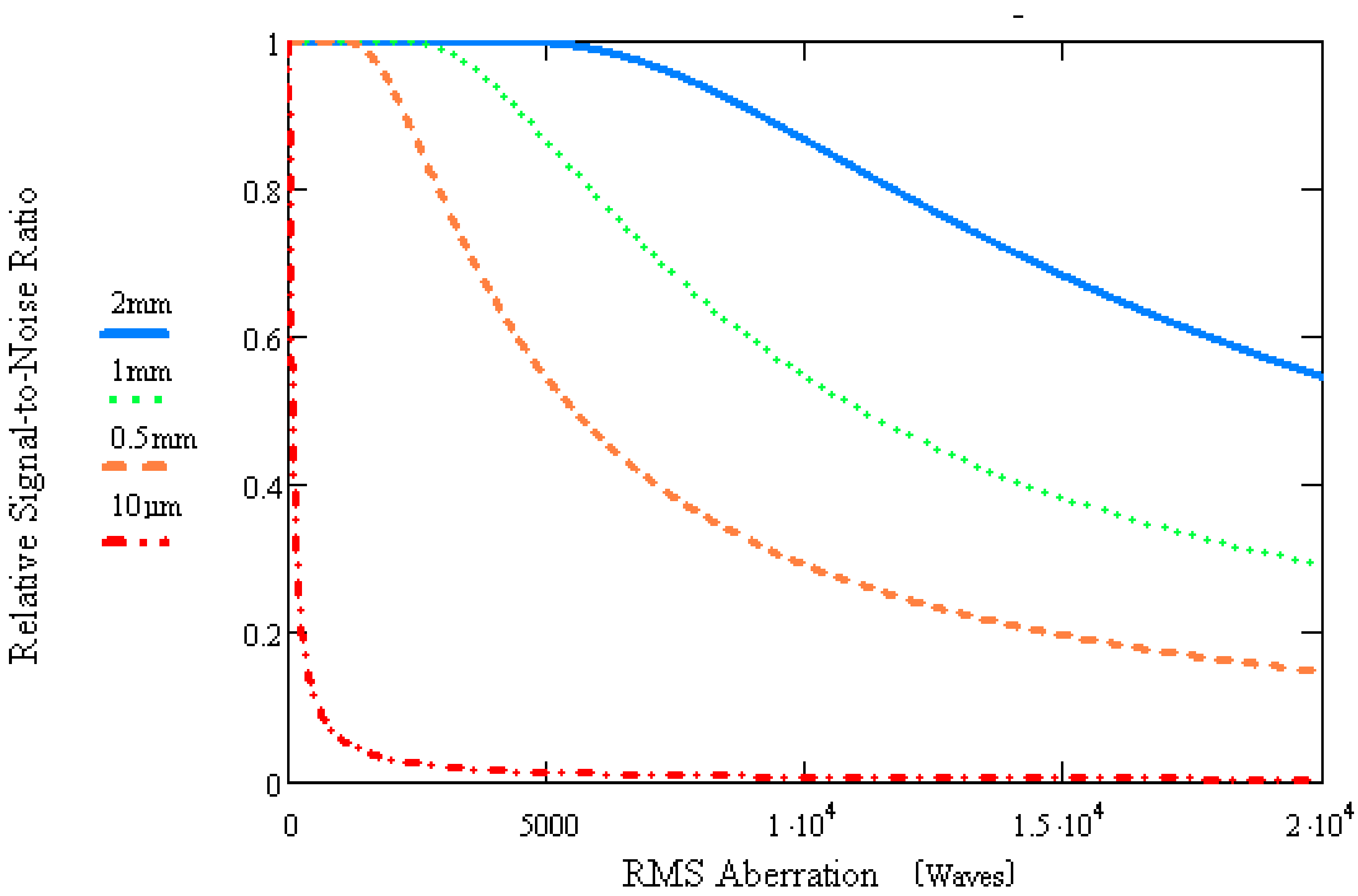
$$SNR = \frac{N_{Star+Sky} - N_{Sky}}{\sqrt{N_{Star+Sky} + N_{Sky} + N_{Detector} + S^2}} \,,$$

where *Ns* are counts and *S* models atmospheric scintillation

- Figures of merit follow for various mirror situations

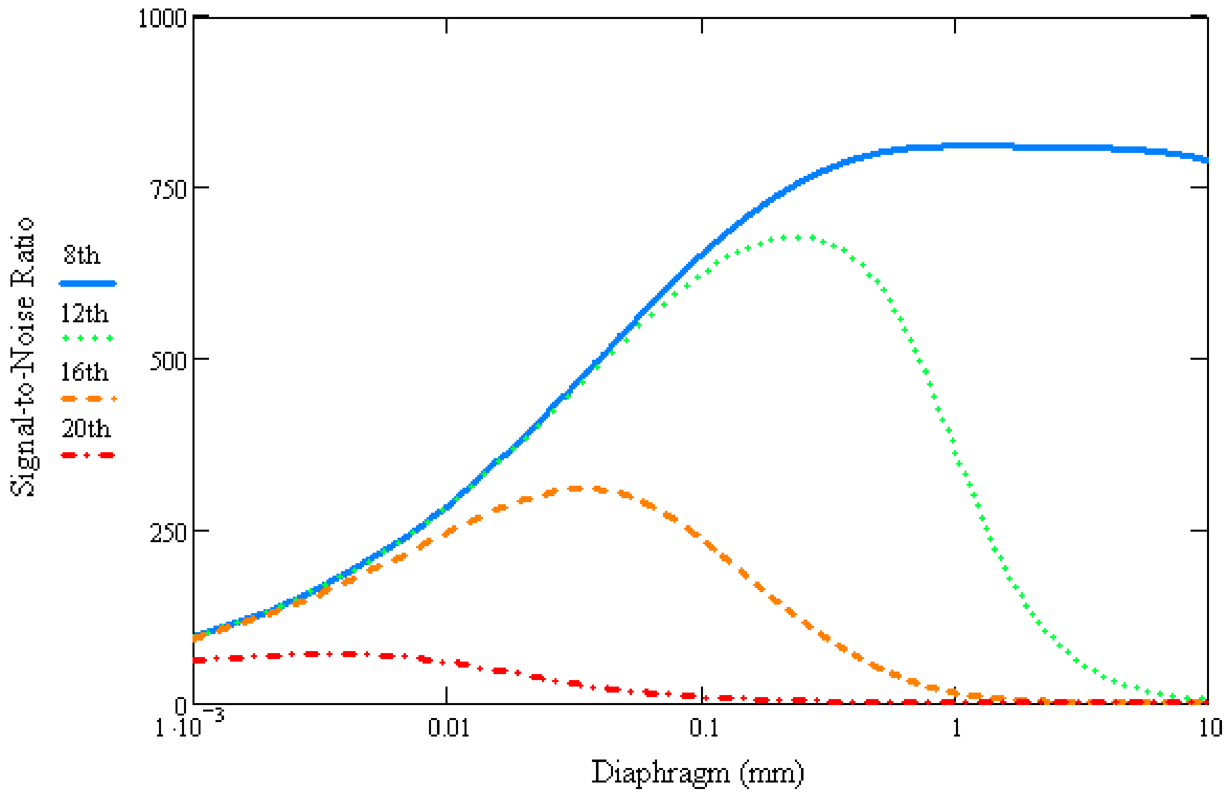
Figures of Merit II

- Random surface height aberrations
- Bright point source
- *f*/1.9, 1.6-m mirror
- Various diaphragms
- Visible light

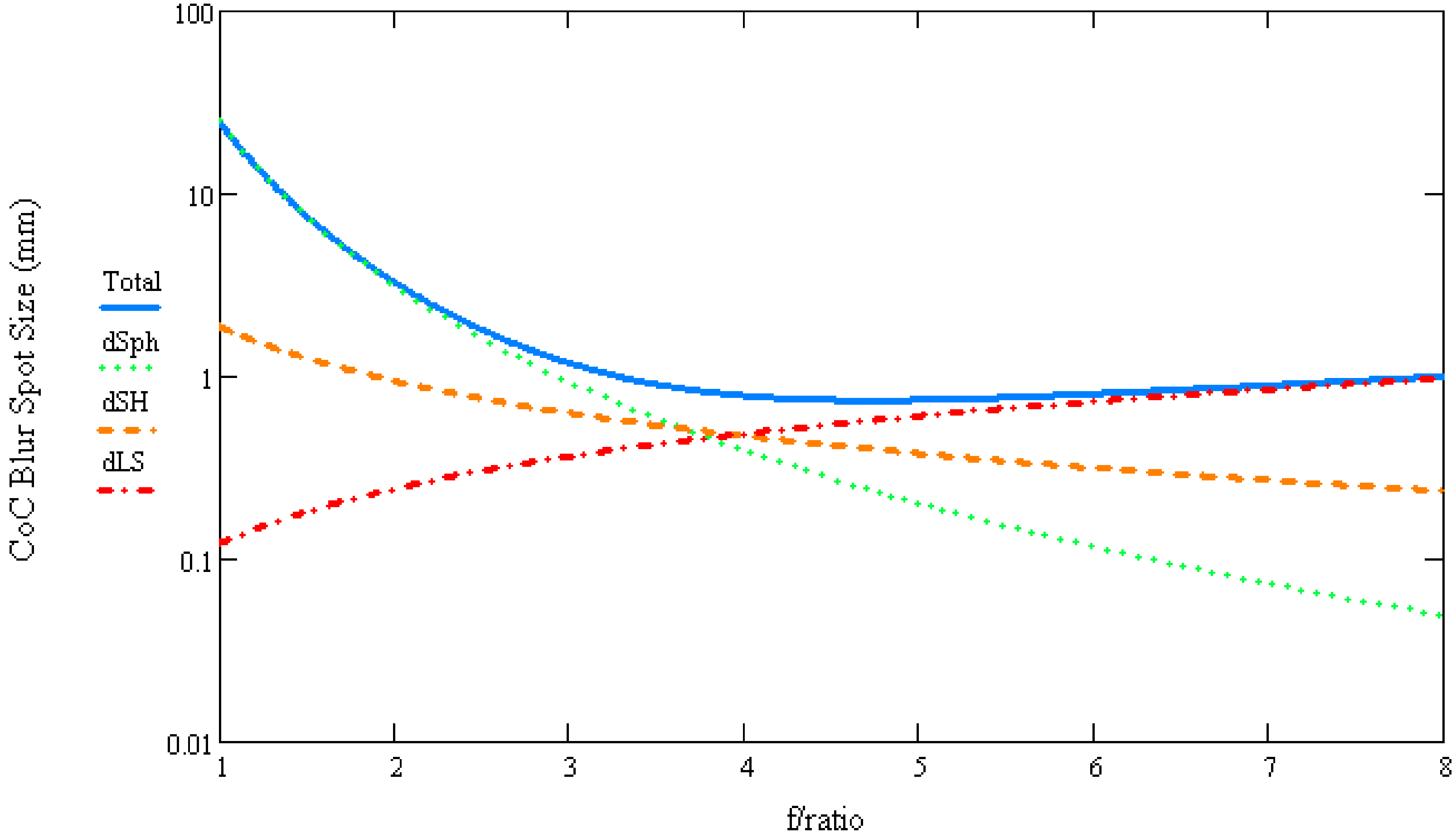


Figures of Merit III

- Local slope aberrations : 10 waves rms gradient norm
- 4 program star cases; *V* = +21 / arcsec squared background
- *f*/1.9, 1.6-m mirror
- Scintillation 1000-m, air-mass 1.5



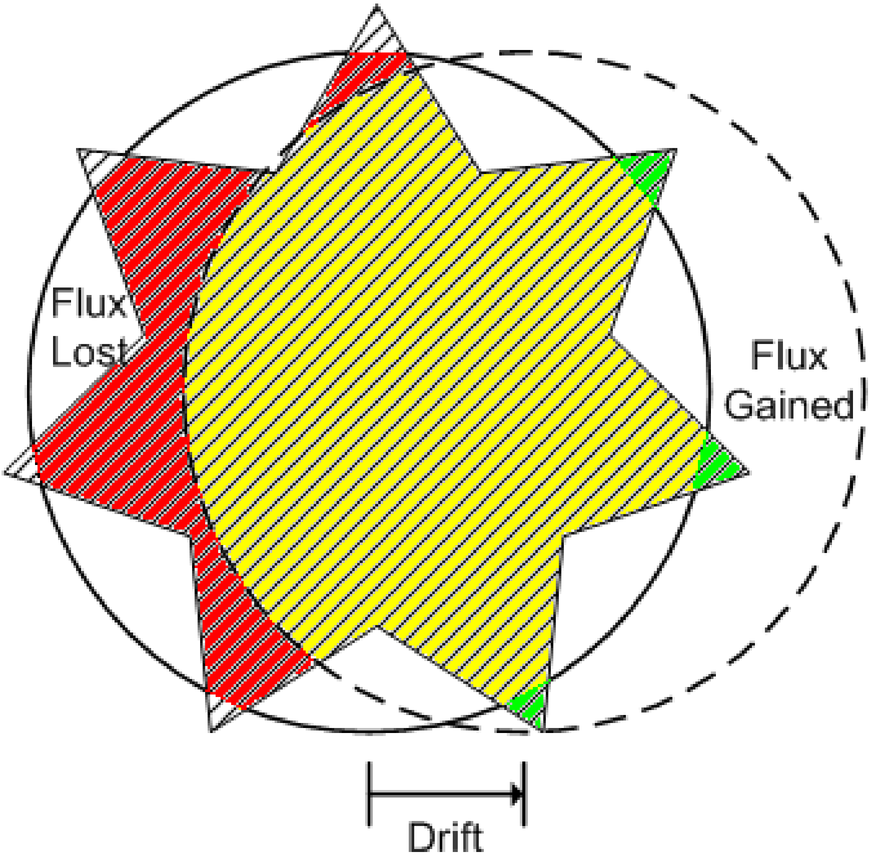
Figures of Merit IV



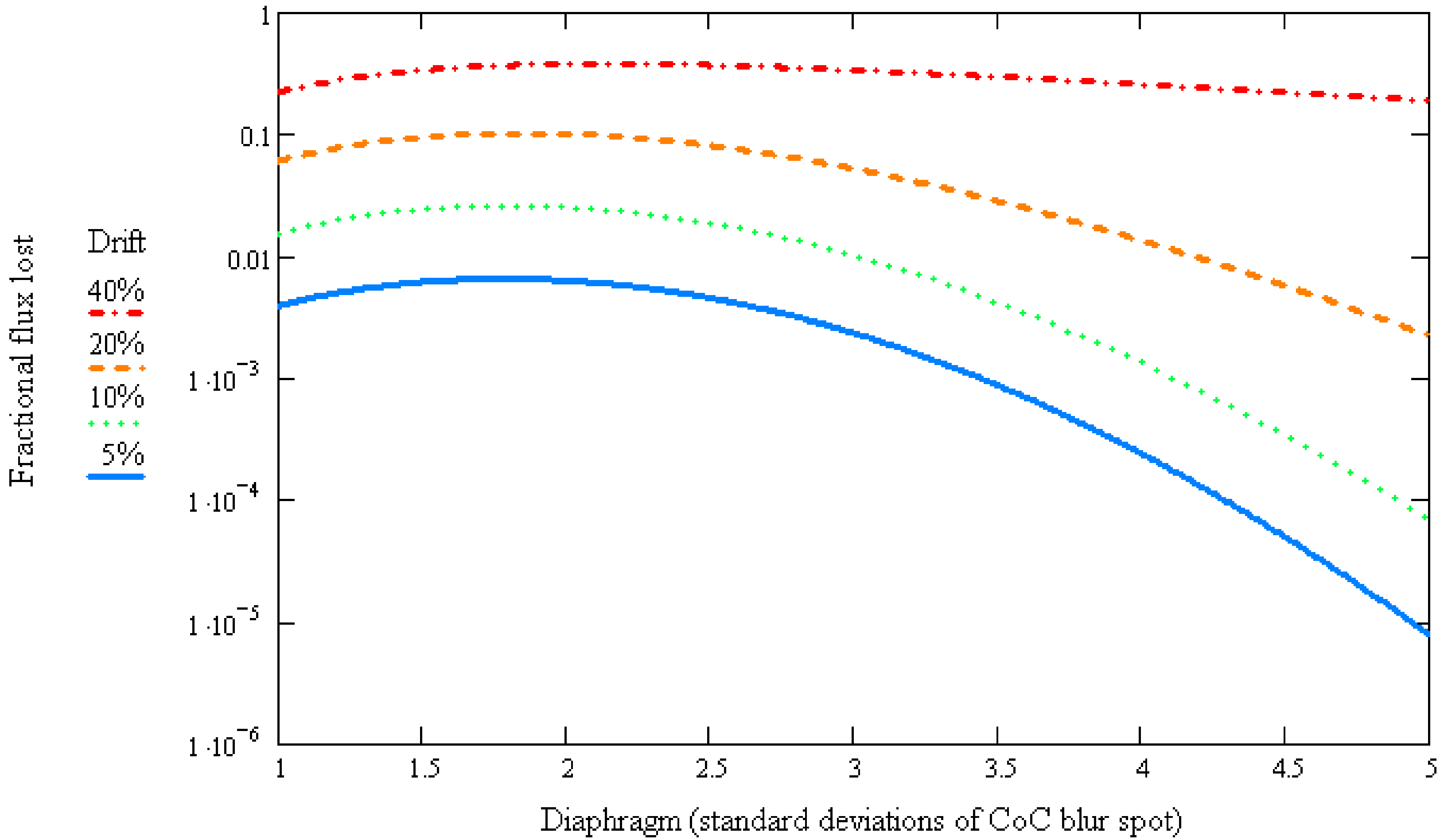
CoC size as a function of *f-ratio*. Spherical, 2500 waves rms surface height, and 10 waves rms gradient norm local slope aberrations are depicted.

Figures of Merit V

- Drifting circular detector diaphragm
  - **Red** – flux lost
  - **Green** – flux gained

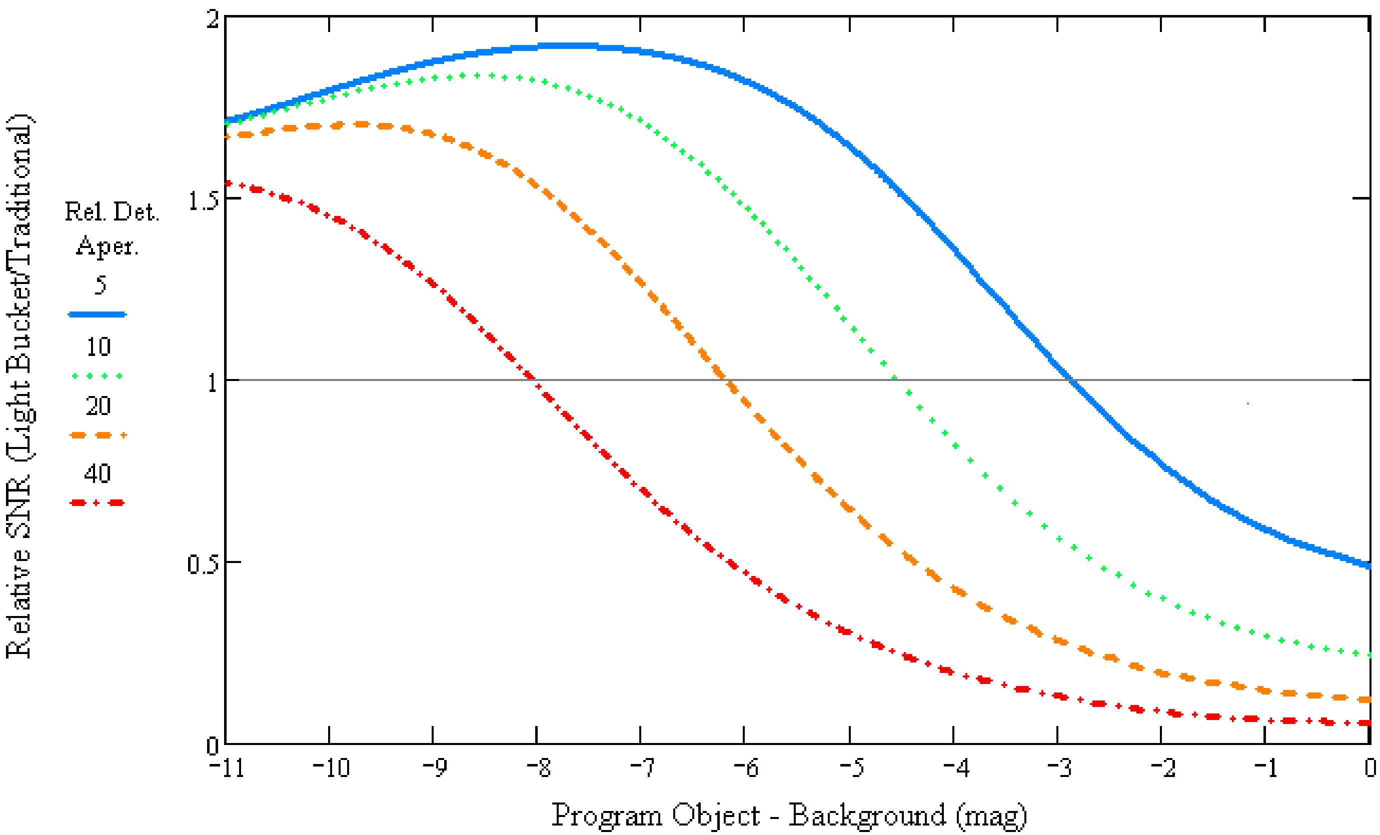


Figures of Merit VI



Light Bucket Selection

- Traditional f/4, 0.75-m mirror
- Light bucket f/2, 1.5-m
- 4 relative diaphragms diameters
- Scintillation at 1000-m, air-mass 1.5



Some Conclusions

We used a statistical approach for light bucket mirror quality analysis: rms local surface height and wavefront gradient norm values. Some conclusions:

- When possible, limit the diaphragm size to improve the SNR, but not so much as to cause significant tracking errors
- For faint objects peak SNR occurs when diaphragms smaller than the size needed to collect 99% of the flux are used
- Light bucket mirrors excel if the program object is bright in comparison to the background

Contact & More Information

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More details see:

2010 Holenstein, B. D., Mitchell, R. J. , and Koch, R. H., *Figures of Merit for Light Bucket Mirrors* in *Lightweight Alt-Az Telescope Developments*, ed. R. Genet, (Payson, AZ: Collins Foundation Press)