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1993 - 1994
Applied Optics or Experimental Astronomy

CALIBRATED ASTRONOMICAL PHOTOELASTIC MODULATOR POLARIMETER AT THE FLOWER AND COOK OBSERVATORY

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ABSTRACT. The PEMP system is designed to measure simultaneously the linear and circular polarization of stellar sources. This is sufficient to determine any partial elliptical polarization present with a precision of about ±0.01%. The essential elements are a photoelastic polarization modulator (time variable linear retarder), a linear analyzer, and a detector of radiant flux. Lock-In Voltmeters are used to generate electrical signals corresponding to the linear polarization flux modulation, the circular polarization flux modulation, and the unpolarized flux. Electrical and polarization calibrations serve to correct for the inherent systematic (bias) errors of the system. All of the electrical signals are interfaced with a microcomputer and software has been developed to generate astrophysically useful polarization data from the measurements.

1. Introduction

Binary and multiple star systems are believed to be the most common sub-systems in the Milky Way Galaxy. Since these are in a great variety of evolutionary states, detailed quantitative knowledge of their astrophysical characteristics will permit verifications of the predictions of current theories of stellar evolution, stellar interiors and atmospheres, nucleosynthesis, magnetohydrodynamics, and relativistic and/or Newtonian dynamics. At Flower and Cook Observatory of the University of Pennsylvania, polarimetry of binary star systems, especially eclipsing ones, has resulted in greater knowledge of the nature and extent of the circumstellar material which envelopes many close binaries. By measuring the linear and/or circular degree of polarization of the received radiant flux as a function of effective wavelength and time (phase locked with the orbital revolution and/or stellar rotation), it is possible many times to ascertain the nature of the scattering material and also its distribution. In recent studies by Holenstein (1991), the physical conditions in the photospheres and atmospheres of some luminous late spectral type variables have been explored by this technique. It should be noted that, to the best of our knowledge, Flower and Cook Observatory and the University of Wisconsin are the only observatories in the U. S. engaged currently in this kind of research.

In many cases the degree of polarization to be measured is less than about .01 (1%). The precision required is about ±.0001 (±.01%). With the development of commercially available photoelastic modulators based on the designs of Jasperson and Schnatterly (1969) and Kemp (1969, 1981), it became possible to achieve this precision. Although we had been engaged in polarimetry of binary star systems since 1973 using other techniques, we initiated the development and use of the present instrument in 1981. Since it was to be used for the specific purpose described above, it may differ from those of others who have pursued other observational programs (Stokes et al, 1976; Kemp et al, 1981), but the essential component is the photoelastic modulator. With the advent of the microcomputer, data acquisition, data processing and operation control became possible directly at the telescope.

The key electro–optical devices used in the polarimeter are the MORVUE, Model PEM–3 Photoelastic Modulator (sinusoidally time variable retarder), a Glan–Thompson linear analyzer and a multiplier photocell (RCA 4509, similar to the commercially available RCA 8645) mounted to rotate together around the optical axis of the telescope. Provision is made for orienting and clamping the modulator and analyzer together so that the fast axis of the first and the transmission axis of the second are at a desired difference of instrumental azimuth, usually ±45°.

The instrument may be afflicted with the following sources of systematic (bias) errors:

a. Non–linearity of the electrical signal at the anode of the detector with respect to the degree of polarization of the incident flux caused by non–linearity of the modulator.
b. The modulator (time variable retarder) and/or the analyzer are not ideal and homogeneous.
c. The peak retardance of the modulator is not known accurately.
d. The relative orientation of the fast axis of the modulator and the transmission axis of the analyzer are not known accurately.
e. The relation between the position angle on the celestial sphere of the electric vector of the stellar polarization and the instrumental azimuth is not known accurately.
f. The rotation axis of the polarimeter may not be parallel to the optical axis of the telescope.
g. The optical elements may not be properly aligned with the rotation axis of the polarimeter.

In order to avoid determining each of the above errors separately, the instrument was designed for the following built–in calibration and/or orientation devices:

a. Means for providing accurately known instrumental sources of partial linear polarization in the range of approximately 5% to 80%.
b. Means for providing an instrumental source of linear polarization with a known orientation of the electric vector. Means, also, for relating this orientation to a position angle on the celestial sphere.
c. Means for adjusting the rotation axis of the polarimeter with respect to the optical axis of the telescope and verifying its alignment.
d. Means for clamping the analyzer in a specific orientation which fixes the direction of the transmission axis with respect to the detector.
e. Means for rotating the modulator assembly around the optical axis and clamping it so that the difference in instrumental azimuth of the fast axis of the retarder and the transmission axis of the analyzer remains always ±45°.

It is also necessary to provide means for electrical calibration in order to determine the relation of the modulation part of the detector signal at the anode to its average value. There is provision for injecting an accurately known sinusoidal voltage of low harmonic distortion (at twice the lock–in reference frequency) and/or an accurately known direct current voltage into the anode circuit of the detector. Since the calibrations of the lock–in and the detector box circuitry are affected by temperature, and since the equipment is subject to a wide range of ambient temperatures in the dome, sensors have been inserted into each to monitor the pertinent temperatures.

In order to maintain the validity of the polarimeter measurements, periodic observations of "polarization standard" stars and "null standard" stars have been made. It should be understood that, as yet, there is no international consensus on a set of stars which may be considered as polarization standards for long periods of time. The observers at Flower and Cook Observatory have adopted a temporary list from a survey of the astronomical literature; there is no assurance that these are constant over long periods of time. Periodic electrical calibrations have also been necessary; these can be accomplished in the daytime without conflicting with the astronomical observations.

2. Theoretical

As guidance in the design of such an instrument, it is instructive to consider the theory of an ideal, homogeneous time variable retarder followed by an ideal homogeneous linear polarizer. The Mueller matrix representation of the ideal system will allow calculation of the instantaneous transmittance of the system as a function of the azimuth of the fast axis of the retarder, the azimuth of the transmission axis of the analyzer, and the instantaneous retardance of the retarder. The matrix equation could be expanded literally to produce a complicated expression or it could be expanded in a Fourier Series with Bessel Function coefficients. We chose to perform the matrix operations step by step, using a spread–sheet program. We have used the notation, definitions and conventions of Shurcliff (1966) throughout. See Figure 1.

The matrix equation which applies follows:

$$\begin{vmatrix} \mathrm{I} \\ \mathrm{M} \\ \mathrm{C} \\ \mathrm{S} \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & (\cos 2\theta)^2 & \cos 2\theta \sin 2\theta & 0 \\ \sin 2\theta & \cos 2\theta \sin 2\theta & (\sin 2\theta)^2 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{D}^2 - \mathrm{E}^2 + \mathrm{G}^2 & 2\mathrm{DE} & -2\mathrm{EG} \\ 0 & 2\mathrm{DE} & -\mathrm{D}^2 + \mathrm{E}^2 + \mathrm{G}^2 & 2\mathrm{DG} \\ 0 & 2\mathrm{EG} & -2\mathrm{DG} & 2\mathrm{G}^2 - 1 \end{Vmatrix} \begin{vmatrix} \mathrm{I} \\ \mathrm{M} \\ \mathrm{C} \\ \mathrm{S} \end{vmatrix}$$

OUT
PUT
STOKES
VECTOR

Ideal Homogeneous
Linear Polarizer

Ideal Homogeneous
Linear Retarder

IN
PUT
STOKES
VECTOR

where:

$\theta \equiv$ azimuth of the transmission axis of a linear polarizer.
(For PEMP, $\theta = \rho + 45°$)
$\rho \equiv$ azimuth of the fast axis of a retarder.
$\mathrm{D} \equiv \mathrm{M} \sin \delta/2$

$\mathrm{E} \equiv \mathrm{C} \sin \delta/2$
$\mathrm{F} \equiv \mathrm{S} \sin \delta/2$
(F is not used for a linear retarder.)
$\mathrm{G} \equiv \cos \delta/2$
$\delta \equiv$ retardance of a retarder.
M ≡ second Stokes Parameter of the normalized fast eigenvector of a retarder.
C ≡ third Stokes Parameter, etc.
S ≡ fourth Stokes Parameter, etc.
(M, C, S correspond to Q, U, V in another notation)
For an ideal homogeneous linear retarder, $\mathrm{M} = \cos 2\rho$, $\mathrm{C} = \sin 2\rho$, $\mathrm{S} = 0$.

We have assumed the following for an ideal time variable retarder:
$\delta = \mathrm{A} \sin(\omega \mathrm{t})$
where:
δ = instantaneous retardance
A = peak retardance
ω = angular frequency ($\omega = 2\pi\mathrm{f}$, f = temporal frequency)
t = time
$\omega\mathrm{t}$ = modulation phase angle
See Figures 2 and 3 for output flux modulation waveforms for selected Stokes Vectors and various values of the pertinent parameters.

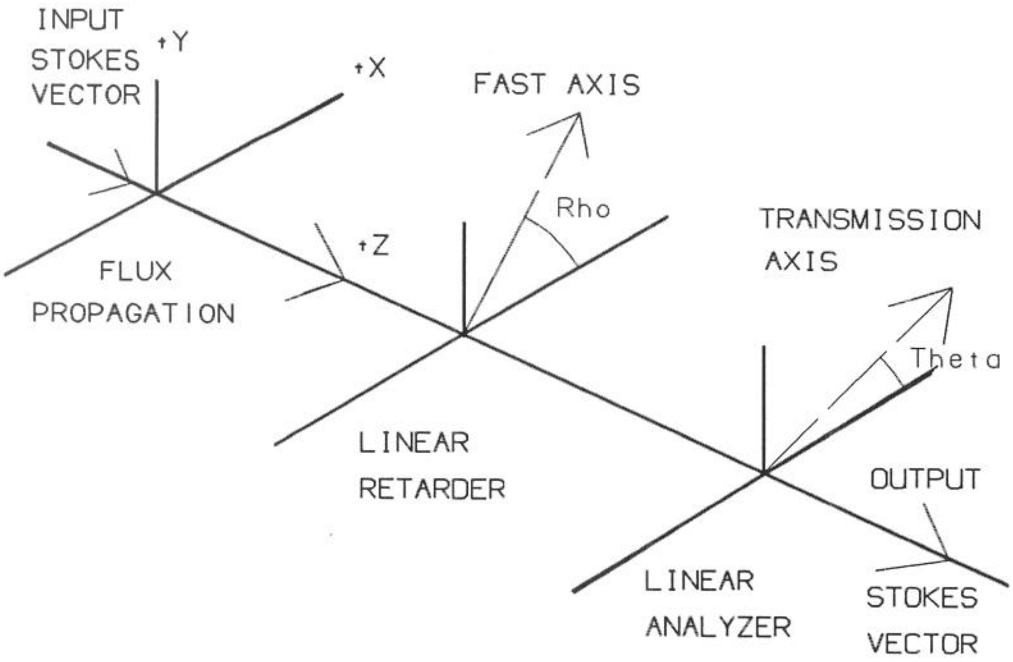

Figure 1. Coordinate system and conventions applicable to PEMP. Shurcliff arbitrarily calls +X, horizontal. All azimuths are with respect to the +X direction in the X–Y plane. $-180° \leq \mathrm{AZIMUTH} \leq +180°$

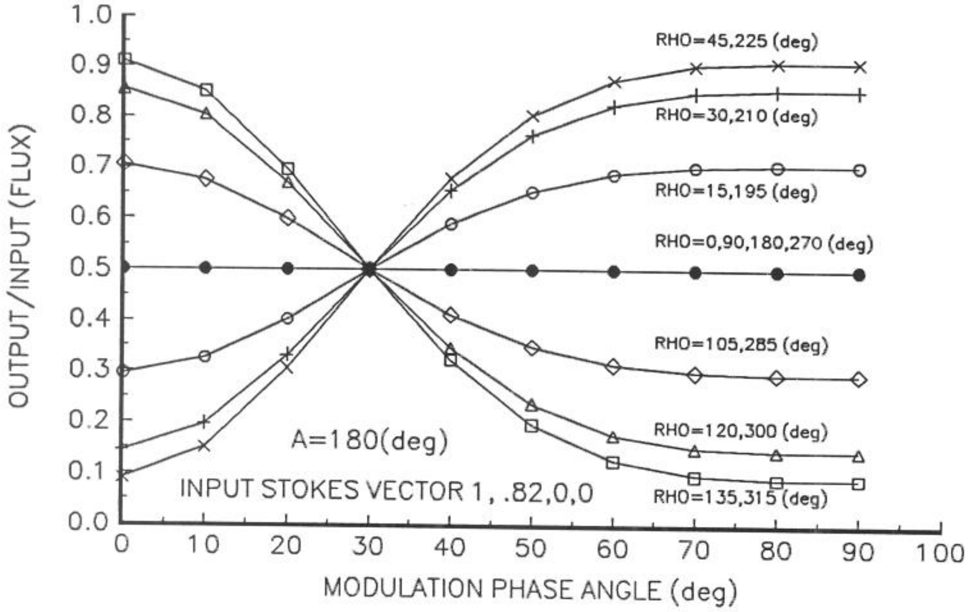

Figure 2. Selected idealized flux modulation waveforms for an 82% linearly polarized input flux with electric vector at azimuth zero degrees. The waveform from phase angle 90 to 180 degrees is the mirror image of that from 0 to 90 and repeats itself from 180 to 360.

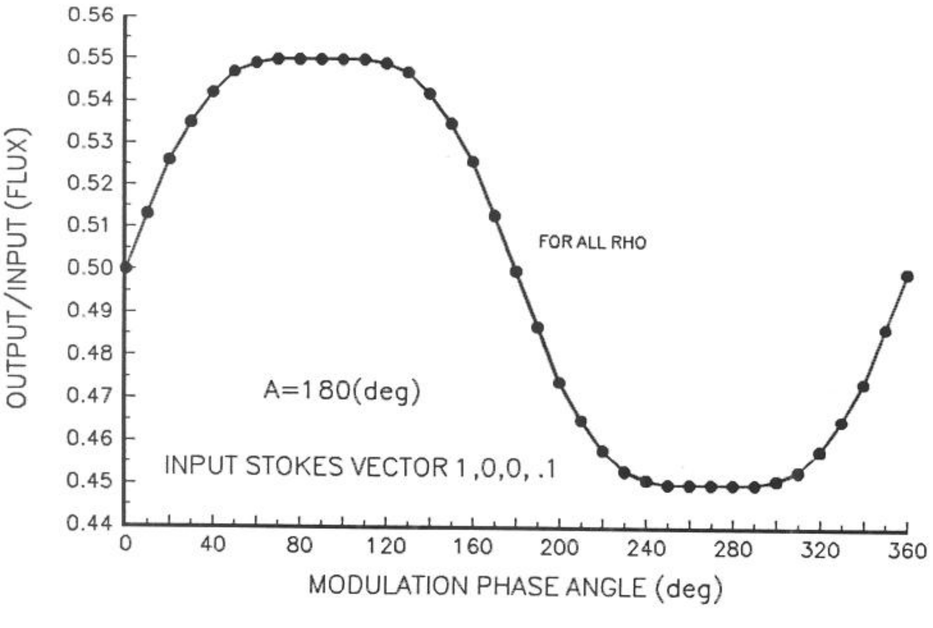

Figure 3. Idealized flux modulation waveform for a 10% right circularly polarized input flux.

3. Optical Components

The optical head (see Figure 4) is attached to the 72cm aperture conventional Cassegrain reflector at the cassegrain focus. It implements the following functions:

a. Acquisition and centering of the selected program star.

b. Separation, by using a small aperture, of the radiant flux of the program star from that of all other stars in the field including the general sky background (a small part of the background near the program star inevitably will reach the detector). The apertures are made of Delrin (a dielectric plastic) in order to reduce any instrumental polarization.

c. Generation artificially of a known linearly polarized radiant flux (degree of polarization and orientation of the electric vector) which can be inserted to irradiate the detector. This is implemented by a green LED whose flux is reflected toward the detector by an uncoated glass surface at an angle of 45 degrees to the optical axis. This reflection causes a linear polarization of about 82% with a known orientation of the electric vector.

d. Wavelength filtering of the stellar flux (intermediate band, FWHM $\sim$ 200Å; broad band, FWHM $\sim$ 1000Å).

e. Collimation of the divergent beam from the cassegrain focus (and the aperture) before it passes through the photoelastic modulator and Glan-Thompson analyzer. This minimizes field of view problems in retarders and polarizers.

f. Incidence of the radiant flux from the program star on the detector as a "Ramsden Disk" which is a real image of the entrance pupil of the telescope. This ensures that the flux pattern on the cathode of the detector is nearly stationary even if the star image drifts in the aperture during an integration. This minimizes the effect of the spatial variation of responsivity of the cathode of the detector.

4. Data Acquisition (see Figure 5)

4.1. BLOCK DIAGRAM

Partially polarized radiant flux from the program star passes through the optical filter, the Photoelastic Modulator, and the Glan–Thompson Analyzer successively and is finally incident on the photocathode of the Multiplier Photocell. The anode output of the multiplier photocell is fed to the following:

Channel #1 — FET high frequency bandpass preamplifier and Ithaco, Mod. 391A Lock–In Voltmeter (set in 2f reference mode). The analog output voltage is the linear polarization "modulation" signal.

Channel #2 —Step variable current gain operational amplifier. The analog output voltage is the average signal related to the unpolarized part of the incident flux.

Channel #3 — The output of the FET preamplifier is also fed to the Stanford Research Systems Mod. SR530 Lock–In Voltmeter (set in 1f reference mode). The output digital voltage is the circular polarization "modulation" signal. The SR530 is directly interfaced to the IBM–AT computer.

The Photoelastic Modulator, the Glan–Thompson Analyzer and the Multiplier Photocell are clamped to each other in a fixed orientation and the whole assembly rotates around the optical axis of the system. The azimuth (AZI) of the assembly is electrically

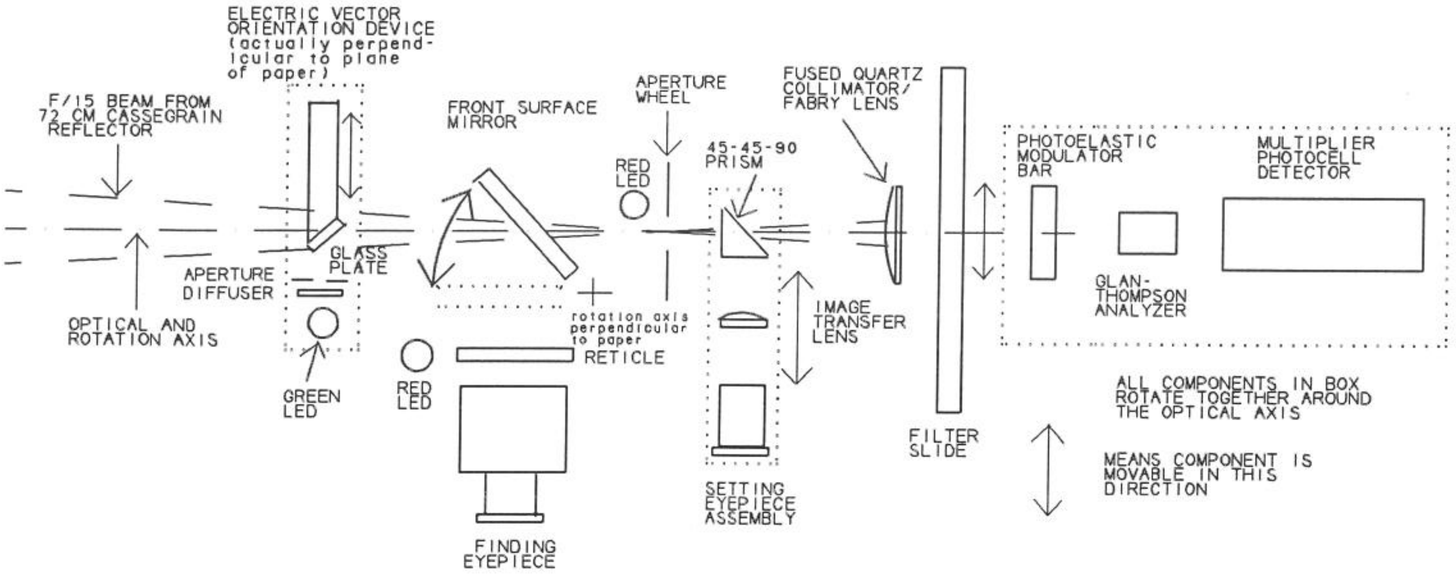

Figure 4. Optical schematic. (Dimensions not to scale)

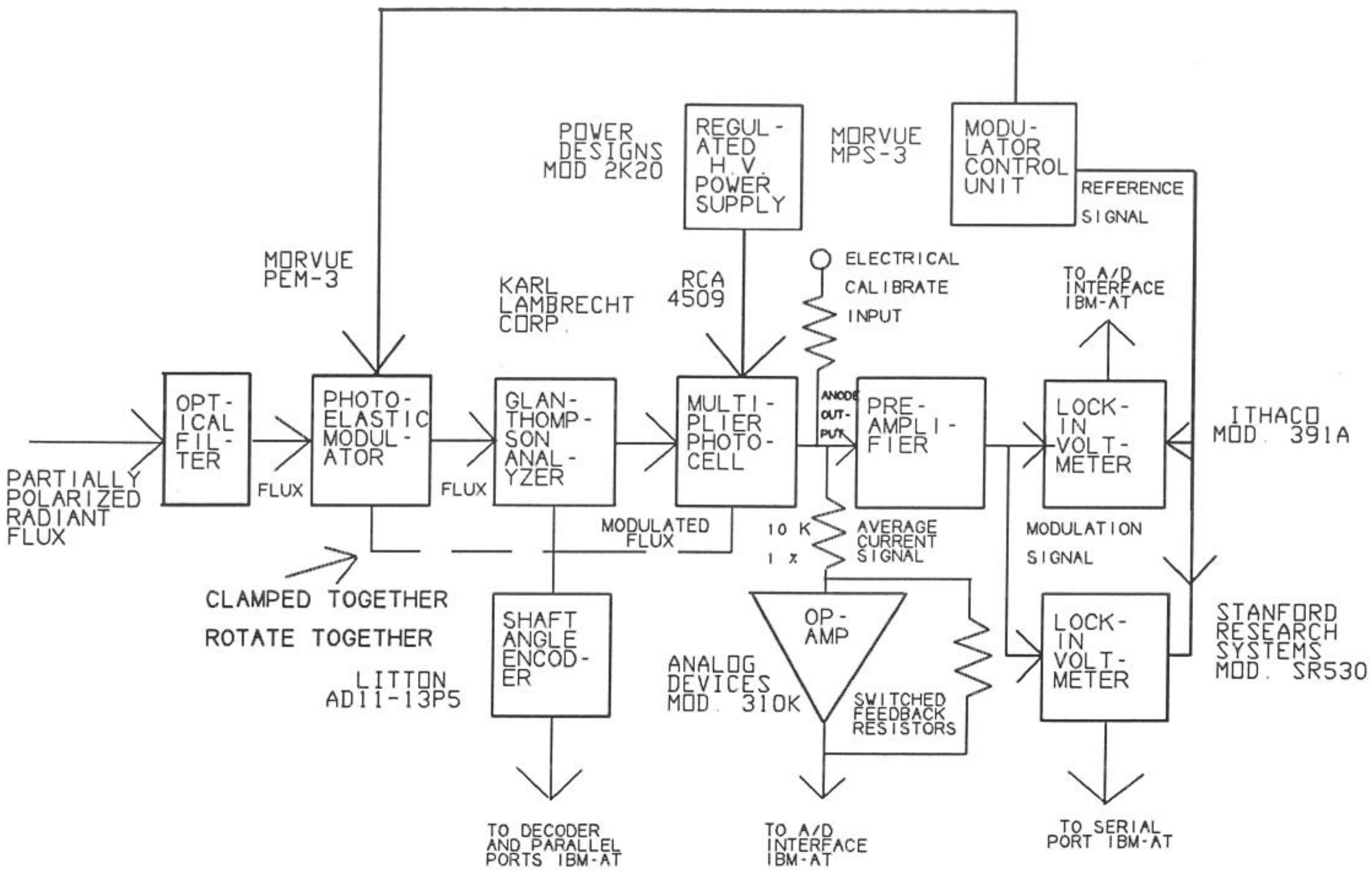

Figure 5. Block diagram of the PEMP system.

encoded by the Shaft Angle Encoder. The various electrical outputs are sent to various ports of an IBM–AT microcomputer. All analog outputs are converted to digital by voltage to frequency converters. The frequency outputs are fed to 32 bit binary up-counters which perform as integrators under the control of the computer through Programmable Peripheral Interfaces which also feed the integrated counts to the IBM-AT data bus.

4.2. OPTIMUM OPERATION

Observational efficiency may be defined as the number of astrophysically useful measurements of a specified accuracy made per unit of observing time. The factors which determine the accuracy in the PEMP system are:

a. The electrical noises of the detector: shot noise caused by the radiant flux of the star, shot noise caused by the background flux, thermionic shot noise, and electrical noise caused by stellar scintillation.
b. Accuracy of the electrical calibration.
c. Accuracy of the polarization calibration.
d. Responsivity of the detector.
e. The total radiant flux collected from the star.
f. Electrical noises inherent in the Lock-Ins (usually negligible).
g. Electrical noises inherent in the rest of the system (usually negligible).

Since its inception, the PEMP system has undergone a continuous theoretical and experimental analysis. The sources and causes of the inherent random (electrical noise) and systematic (bias) errors have been investigated. Computer program ASTRAD (Blitzstein *et al*, 1987) has been used to simulate the system. In this way, we have found that scintillation noise predominates in the output of the detector for stars to about Johnson V magnitude, +9. Various experimental and/or observational tests have helped to determine quantitatively the accuracy and/or precision of the directly measured quantities and also that of the astrophysically useful quantities deduced from them.

These studies have led to the adoption of a standard observing procedure which provides the greatest efficiency subject to the practical observational limitations. The following rules have been adopted:

a. Integration times for observation of the flux of the star plus the background shall be 90 seconds.
b. Integration times for the background alone shall be 30 seconds.
c. Ten sets of readings shall be made at approximately 36 degree intervals of azimuth (AZI) from 0 to about 324 degrees.

In this way, a complete determination of the desired quantities can be accomplished in about 26 minutes, on the average, including all integrations and any "housekeeping" time necessary.

5. Data Reduction

5.1. ALGORITHMS,LINEAR POLARIZATION

The output of the Lock–in Voltmeter (Flux modulation, Channel #1) and the operational amplifier (Average flux, Channel #2) may be represented by a truncated Fourier series whose argument is the instrumental azimuth angle read by the shaft angle encoder.

$\mathrm{NET1} = (\mathrm{S1} + \mathrm{B1}) - \mathrm{B1}$
$\mathrm{NET2} = (\mathrm{S2} + \mathrm{B2}) - \mathrm{B2}$
AZI = instrumental azimuth angle
S1 = integrated counts, star, in Channel 1
S2 = integrated counts, star, in Channel 2
B1 = integrated counts, background, in Channel 1
B2 = integrated counts, background, in Channel 2
(S1,S2,B1,B2 include thermionic emission or "dark" current)

It was assumed, a priori, that:

$$\begin{aligned}\frac{\mathrm{NET1}}{\mathrm{NET2}} &= \mathrm{A} + \mathrm{B}\sin(\mathrm{AZI}) + \mathrm{C}\cos(\mathrm{AZI}) + \mathrm{D}\sin 2(\mathrm{AZI}) + \mathrm{E}\cos 2(\mathrm{AZI}) \\ &\quad + \mathrm{F}\sin 3(\mathrm{AZI}) + \mathrm{G}\cos 3(\mathrm{AZI}) + \mathrm{J}\sin 4(\mathrm{AZI}) + \mathrm{H}\cos 4(\mathrm{AZI}) \\ &\quad + \text{higher order terms,}\end{aligned}$$

where A, B, C, etc. are coefficients to be determined.

The observed NET1/NET2 are fitted to a truncated series, as above, for the 10 AZI's by the method of Least Squares. Obviously, no more than 9 terms can be included. An experimental system error analysis has shown that the terms in 3(AZI) and 4(AZI) represent only the effects of random noises in the signals (Blitzstein, 1991; Elias, 1990). Then, only the coefficients A, B, C, D, and E are consequences of the information in the partially polarized radiant flux from the program star. D and E are the measure of the partial linear polarization. See Figure 6 for an example of least squares fitting to the observations.

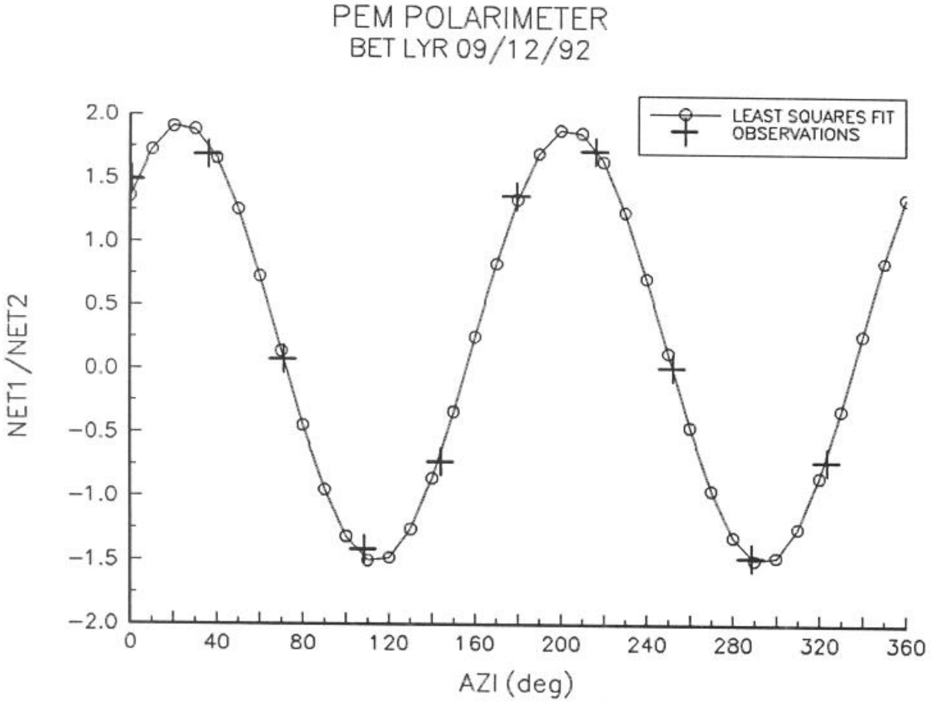

Figure 6. Example of a least squares fit of a truncated Fourier series to the observations of Beta Lyrae.

5.1.1. *Peak Modulation.*

$$D = N \sin 2\,(\mathrm{AZI}_{\mathrm{peak}})$$

$$E = N \cos 2\,(\mathrm{AZI}_{\mathrm{peak}})$$

$$N \equiv \sqrt{D^2 + E^2} = \text{PEAK MODULATION}$$

5.1.2. *Azimuth of the First Negative Peak.*

$$\text{AZI FIRST NEG. PEAK} = 1/2 \arctan \frac{D}{E} + \text{multiple of } 90^\circ$$

If the electrical signs of the Ithaco Lock–In outputs are used for D and E, the following rules are used for determining AZI FIRST NEG. PEAK:

ELECTRICAL SIGNS	QUADRANT 2(AZI PEAK)	SIGN AZI PEAK CALCULATED	RANGE AZI PEAK	ADD TO AZI PEAK CALCULATED
D = + E = +	3	+ (range 0° to 45°)	90° to 135°	90°
D = + E = −	4	− (range − 45° to 0°)	135° to 180°	180°
D = − E = −	1	+	0° to 45°	0°
D = − E = +	2	−	45° to 90°	90°

5.1.3. *Natural Position Angle of the Electric Vector of Stellar Source.*

$$\mathrm{PA} = 90^\circ + (\text{AZI FIRST NEG PEAK}) - (\text{AZI FIRST NEG PEAK})_{\text{EV SOURCE}}$$

$0 \leq \mathrm{PA} \leq 180^\circ$; In polarization determinations, PA is ambiguous by 180°.
(The position angle of the electric vector of the EV source is set to $90^\circ \pm \frac{1}{4}^\circ$ using a sight which is attached to the Optics Box for viewing a star image which is offset from the optical axis at a position angle of precisely 90°. This is accomplished by acquiring a bright star in one of the apertures and then driving the telescope in hour angle, only, until the star is seen on the knife edge of the sight.)

5.1.4. *Natural Polarization.* Adopted Linear Polarization Parameter definition:

$$\mathrm{WP} \equiv 1.031\, \frac{C_1\, N}{C_2}$$

1.031 is a theoretical number which was, in early development, believed to be significant. Although no longer believed significant, it is retained for consistency with earlier definitions. C_1 is the electrical calibration constant for Channel #1, and C_2 is for Channel #2 (see section 6.).

To account for the non-linearity of the modulator, we have assumed:

$$\frac{\mathrm{V}}{(\mathrm{WP})} \equiv \mathrm{L} + \mathrm{M}\,(\mathrm{WP}) \qquad (\text{Assumes } \mathrm{V} = 0 \text{ if } \mathrm{WP} = 0)$$

where:
V = the degree of polarization (Shurcliff notation)
(NATURAL POLARIZATION) = 100 V, units percent.
The values,
$\mathrm{L} = 1.139, \quad \sigma(\mathrm{L}) = \pm .009$
$\mathrm{M} = -0.0845, \quad \sigma(\mathrm{M}) = \pm .02$
have been adopted, based on the Hybrid (standard stars and artificial sources) Polarization Calibration of 03/23/85 (see section 7.1).

5.1.5. *Natural Q, Natural U.*

$\mathrm{Q} = (\text{NATURAL POLARIZATION}) \cos 2(\mathrm{PA})$ (Shurcliff notation, M)
$\mathrm{U} = (\text{NATURAL POLARIZATION}) \sin 2(\mathrm{PA})$ (Shurcliff notation, C)

5.1.6. *Probable Error of the Azimuth of the First Negative Peak*

$$\text{P.E. AZI FIRST NEG PEAK} = \frac{57.^{\circ}296}{2|\mathrm{E}|\left[1+\left(\frac{\mathrm{D}}{\mathrm{E}}\right)^2\right]}\sqrt{\mathrm{PE}^2\,(\mathrm{D}) + \left(\frac{\mathrm{D}}{\mathrm{E}}\right)^2 \mathrm{PE}^2\,(\mathrm{E})}$$

where:
PE(D) and PE(E) are from the least squares fit to NET1/NET2.
(The probable error of the natural position angle of the electric vector is assumed to be the same as that for AZI FIRST NEG PEAK)

5.1.7. *Probable Error of Natural Polarization.*

$$\text{P.E. (NATURAL POLARIZATION)} = 0.6745\,(100)\,\sigma(\mathrm{V}), \text{ units percent}$$

$$= 67.45\sqrt{(\mathrm{WP})^2\sigma^2(\mathrm{L}) + [\mathrm{L} + 2\mathrm{M}(\mathrm{WP})]^2\,\sigma^2(\mathrm{WP}) + (\mathrm{WP})^4\sigma^2(\mathrm{M})}$$

where:
$\sigma(\mathrm{L})$ and $\sigma(\mathrm{M})$ are from the least squares fit to the data of the Hybrid Polarization Calibration.

$$
\begin{aligned}
\sigma(\mathrm{L}) &= \pm.009 \\
\sigma(\mathrm{M}) &= \pm.02 \\
\sigma(\mathrm{WP}) &= 1.031 \sqrt{\frac{\mathrm{N}^2}{\mathrm{C_2}^2}\,\sigma^2\,(\mathrm{C_1}) \;+\; \frac{\mathrm{C_1}^2}{\mathrm{C_2}^2}\,\sigma^2\,(\mathrm{N}) \;+\; \frac{\mathrm{C_1}^2\mathrm{N}^2}{\mathrm{C_2}^4}\,\sigma^2\,(\mathrm{C_2})} \\
\sigma^2(\mathrm{N}) &= \frac{\mathrm{D}^2\sigma^2(\mathrm{D}) \;+\; \mathrm{E}^2\sigma^2(\mathrm{E})}{\mathrm{D}^2 \;+\; \mathrm{E}^2}
\end{aligned}
$$

(σ(D) and σ(E) are from the least squares fit to NET1/NET2).

$$
\begin{aligned}
\sigma(\mathrm{C_1}) &\approx .012\mathrm{C_1} \\
\sigma(\mathrm{C_2}) &\approx .004\mathrm{C_2}
\end{aligned}
$$

(from the least squares fits to the electrical calibrations).

5.1.8. *Probable Error of Natural Q and U.*

NP $\equiv$ natural polarization   PA = natural position angle

$$
\mathrm{PE(Q)} \;=\; \sqrt{\cos^2 2\mathrm{PA} \times \mathrm{PE}^2\;(\mathrm{NP}) \;+\; 4(\mathrm{NP})^2 \sin^2 2(\mathrm{PA})\;\mathrm{PE}^2\;(\mathrm{PA})}
$$

$$
\mathrm{PE(U)} \;=\; \sqrt{\sin^2 2(\mathrm{PA}) \times \mathrm{PE}^2\;(\mathrm{NP}) \;+\; 4(\mathrm{NP})^2 \cos^2 2(\mathrm{PA})\;\mathrm{PE}^2\;(\mathrm{PA})}
$$

5.2 ALGORITHMS, CIRCULAR POLARIZATION

$$
\mathrm{V} \;=\; \frac{\mathrm{C_3A}}{\mathrm{C_2}\overline{\mathrm{NET2}}}\,\frac{\Pi_{\mathrm{star+background}}}{60}
$$

where:

V = Circular Polarization Stokes Vector in %.
$\mathrm{C_3}$ = The Circular Polarization Calibration Parameter (units, microvolt per volt).
A = The constant coefficient in the Least Sqares fit of a truncated Fourier Series to the output of the Stanford Research Systems, SR530 (units, volts).
$\mathrm{C_2}$ = The Channel 2 linear polarization electrical calibration constant.
$\overline{\mathrm{NET2}}$ = Arithmetic Mean of the NET2 integrated counts.
$\Pi_{\mathrm{star+background}}$ = Integrating time in seconds.
60 = 60 seconds.

The estimated error reported for the circular polarization is:

$$
\sigma_{\mathrm{V}} \;=\; \frac{\Pi_{\mathrm{star+background}}}{60\mathrm{C_2}\overline{\mathrm{NET2}}} \sqrt{(\mathrm{A}^2\sigma^2_{\mathrm{C_3}} + \mathrm{C}_3^2\sigma^2_{\mathrm{A}}) \;+\; (\mathrm{C_3A})^2 \left(\frac{\sigma^2\mathrm{C_2}}{\mathrm{C}_2^2} \;+\; \frac{\sigma^2_{\overline{\mathrm{NET2}}}}{(\overline{\mathrm{NET2}})^2}\right)}
$$

where $\sigma_{\mathrm{C_3}}$ is determined from observations of standardized sources of circular polarization, σ_{A} is determined from the least squares solution, $\sigma_{\mathrm{C_2}}$ is determined from the least squares fit to electrical calibration data for Ch.# 2, and $\sigma_{\overline{\mathrm{NET2}}}$ is determined by $\sqrt{\overline{\mathrm{NET2}}}$.

5.3 DATA PROCESSING/SOFTWARE

Several programming languages have been used in the past for polarization data reduction software. We have found that Microsoft Quick Basic Version 4.5 is the most convenient and flexible for our purposes. The speed of operation permits a single set of 10 observations to be processed in about 10 seconds after the last measurement. The observer can monitor the quality of the observations almost in real time. The observations and the data reductions are stored in archival files for future analysis.

6. Electrical Calibrations

Electrical calibrations are necessary to determine the relation between the integrated counts in the channels and the potential difference (voltage) across the precision 10,000 ohm resistor in the anode circuit of the multiplier photocell. Since the system characteristics are functions of ambient and/or internal temperatures, the calibrations are also functions of temperature. Temperature sensors are placed in the Channel 1 lock–in voltmeter and the electronics box of the polarimeter. Electrical calibrations are performed frequently during the year for a range of ambient temperatures in the dome. The dependence of the calibration constants C_1 and C_2 on temperature for each sensitivity step of channels 1 and 2, respectively, is determined by a linear least squares fit. The temperature sensor analog outputs are sent to an interface board and converted to counts in the IBM–AT computer. The current electrical calibration least squares fits are incorporated into the software, PEM Polarimeter Reduction.

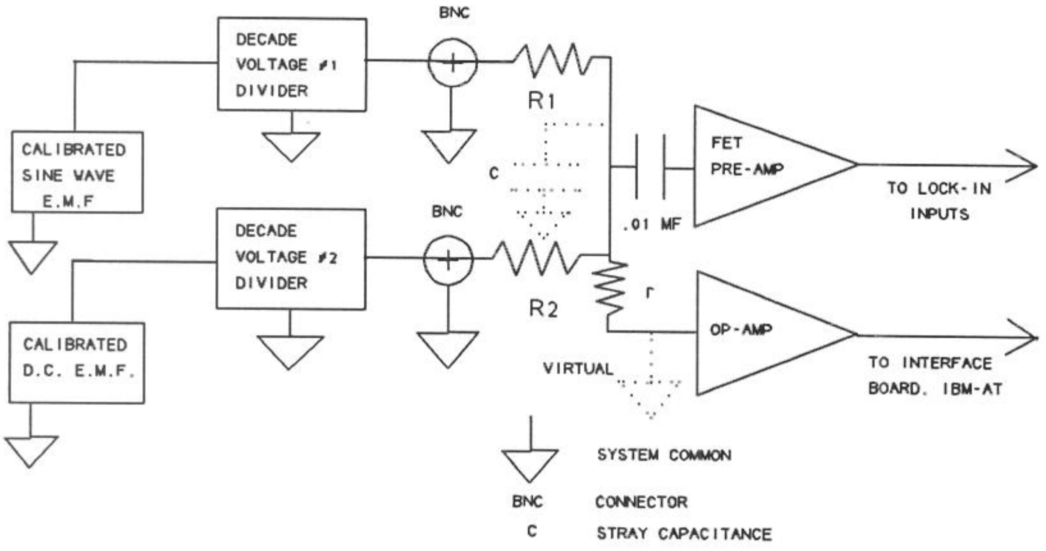

Figure 7. Schematic diagram of the electrical calibration circuitry.

Figure 7 is a schematic diagram of the electrical calibration circuitry. The values of the resistors R1, R2, and r are measured carefully in the laboratory. The output of the sine wave oscillator is measured by a precision A.C. digital voltmeter and the D.C. E.M.F. (battery) is measured by a precision D.C. voltmeter. The effects of stray capacitance at the input to the FET preamplifier have been measured and removed from the linear calibration at the critical frequency for the lock–ins, 100 kilohertz.
From electrical network theory,

$$\mathrm{V_r} = \frac{\mathrm{r}}{1+\frac{\mathrm{r}}{\mathrm{R_1}}+\frac{\mathrm{r}}{\mathrm{R_2}}}\left[\frac{\mathrm{F_1 V_{rms}}}{\mathrm{R_1}} + \frac{\mathrm{F_2 V_{D.C.}}}{\mathrm{R_2}}\right]$$

(the internal impedances of the decade voltage dividers are $<$ 200 ohms in the ranges used and are neglected here)
where:

$\mathrm{V_r}$ = the voltage across resistor r.
$\mathrm{F_1}$, $\mathrm{F_2}$ = the attenuation settings of the precision decade voltage dividers (range .0001 to 1.0000).
$\mathrm{V_{rms}}$ = the root mean square voltage of the sine wave oscillator at 100 kilohertz frequency.
$\mathrm{V_{D.C.}}$ = the voltage of the battery used for the D.C. source.

From the measured effect of stray capacitance across r,

$$\frac{\mathrm{V_r(100kHz)}}{\mathrm{V_r(low\ frequency)}} = .927.$$

The electrical calibration constants are defined as:

$$\mathrm{C_1} \equiv \frac{\mathrm{V_r(100kHz,\ peak,\ Ch.\#1,\ \mu V)}}{\text{integrated count in 60 seconds}}$$

$$\mathrm{C_2} \equiv \frac{\mathrm{V_r(D.C.,\ Ch.\#2,\ \mu V)}}{\text{integrated count in 60 seconds}}$$

where $\mathrm{V_r}$ is calculated from the measured quantities using the above algorithms.

7. Polarization Calibrations

7.1. LINEAR

The system output is non–linear with the degree of polarization of the incident stellar flux. Specifically, the lock–in output voltage is non-linear with respect to the degree of polarization of the flux incident on the photoelastic modulator. Then, polarization calibrations are a necessity for both linear and circular polarization.

Initial attempts at such calibrations involved a pair of artificial instrumental sources. For large degrees of polarization, the Electric Vector Orientation Device (EV) was used since it was necessarily present for orientation purposes also. For small degrees of polarization, two sources were implemented, both of which were found unsatisfactory for calibration purposes for various reasons. All artificial calibration devices involve artificial flux sources, specular reflection from a plane dielectric, and passage of the reflected uncollimated beam through the apertures, modulator, filters, collimator (quartz field or Fabry lens) and Glan–Thompson analyzer to the detector.

Null artificial sources are very difficult to achieve. There are few sources, natural or artificial, which can be certified as having less than 10^{-4} degree of polarization (0.01%)!

Preliminary attempts at polarization calibration with artificial sources, led to the understanding that the following assumptions were sufficient for the purpose:

a. Assume zero output for incident completely unpolarized flux (observation has shown that the instrumental polarization is smaller than the mean random error of the measurements).
b. Assume that the lock-in output is a quadratic function of the input degree of polarization of the form:

$$V = L(WP) + M(WP)^2 \qquad \text{or}$$

$$\frac{V}{WP} = L + M(WP)$$

where:

$WP \equiv$ the linear polarization parameter,
$L \equiv$ the intercept of a linear regression,
$M \equiv$ the slope of a linear regression,
$V \equiv$ the degree of polarization.

Since the attempts to use instrumental artificial polarization sources in the range 0 to ≈ 3% were unsuccessful and since Robert H. Koch had made a series of observations of so-called "polarization standards", including "null standards", it was decided to implement a HYBRID POLARIZATION CALIBRATION which used stellar sources of supposedly known polarization for those in the range 0.% to ≈ 6.% and the EV source at ≈ 82%.

The adopted polarization percentages and position angles of the electric vector and/or Stokes Parameters were averages of polarizations reported in the literature from various observatories for stars which had not been found variable over a decade. It is questionable whether either the interstellar or the intrinsic polarization associated with any star is constant for any long period of time. The EV source was measured a number of times using the Glan–Thompson Analyzer **only** with the modulator **off**. In this way, it was determined that the degree of polarization was very close to 0.820 (82.0%). This agrees well with the theoretical value for reflection from ordinary crown glass of index of refraction 1.52 to 1.54 at an angle of exactly 45 degrees.

Least squares was used to find all the pertinent parameters of the calibration and the Hybrid Polarization Calibration was adopted 03–25–85 as:

$$\frac{V}{(WP)} = 1.139 - 0.0845(WP)$$

7.2. CIRCULAR

Since there are no known standard sources of circular polarization, it was necessary to make the calibration dependent on the linear calibration and an artificial source of circular polarization. This source used a rotatable achromatic quarter wave retarder in conjunction with the EV source described above to produce various degrees of circular polarization incident on the photoelastic modulator and traversing the linear analyzer to the detector. The standardization of the degree of circular polarization (in percent) has been implemented mainly using the EV source and an achromatic quarter wave retarder (Pancharatnam 1955) whose central wavelength is ~ 0.56 μm and whose range of achromatism is ~ .48 to ~ .64 μm (the light source is a green LED).

A rotation on the Poincaré Sphere by a retarder must obey the invariance relation:

$$\mathrm{Q_i^2 + U_i^2 + V_i^2 \; = \; Q_f^2 + U_f^2 + V_f^2}$$

where the subscripts i and f denote the initial and final orientations. Thus, in principle, measuring the linear polarization of a <u>null</u> circular polarization source before and after insertion of a quarter wave retarder at a specific orientation enables the determination of the circular polarization produced and a resulting standardization. This was accomplished by using the EV source and various known non-null linearly polarized stellar sources. This technique is not ideal since it depends on the calibration of the linear subsystem and also on the optical properties of the retarder used. Experience has shown that the available quarter wave plate partially analyzes the incident flux by about 0.05%.

A resolution of IAU Colloquium 23 defined the rotation of the electric vector for <u>positive</u> circular polarization to be <u>counter–clockwise</u> when the detecting system <u>faces</u> the source. Care must be taken when interpeting data from other researchers, since a variety of conventions may be found in the older literature. For example, this sign convention is consistent with Kemp and Wolstencroft (1972), but is the reverse of the sign convention by Shurcliff (1966), which is quite commonly used by most optical engineers and physicists. The sign of the subsystem was verified by use of a known circular polarizer and Mueller Calculus.

8. Summary

8.1. OBSERVATION STATISTICS

Linear polarization measurements were initiated using the PEMP system during August, 1984. Circular polarization measurements simultaneous with linear ones were initiated during November, 1986. Up to October, 1992, 1114 linear only and 2272 elliptical measures of polarization have been made. The following tabulates the numbers of observations by classes of objects:

Close Binary Systems	1673
Luminous Late–Type Variables	621
Beta Cephei Variables	263
Polarization Standards	772
Miscellaneous	57
Total	3386

8.2. EFFICACY OF THE SYSTEM

By analyzing NET1/NET2 by least squares, the effect of variations of the received flux during a run caused by source and/or extinction variations is minimized. When there is strong moonlight, the background is partially linearly polarized. The effect of any polarization of the background is removed during the data processing.

One source polarization determination (linear and circular simultaneously) requires about 26 minutes which includes integration time and data reduction time.

By experience, we have found that the limiting magnitude for a probable error of ±.01% is about Johnson $V = +7$ for an A0 star. The calibrations described above and careful standardization of the observations make us confident that the final results are quite free of systematic (bias) errors.

Our experience has been that the system is very reliable. Steps have been taken to protect the equipment from extremes of temperature and humidity. We estimate the down time due to equipment failures as 0.1% of the available observing time.

9. Contributors

William Blitzstein was and is presently responsible for the mechanical, optical and electronic design and development of the PEMP System. W.B. is also responsible for the on–going systems analysis, including algorithms for data reduction, calibration techniques and experimental and theoretical error analyses.

Robert H. Koch was and is presently responsible for overall supervision of the observational programs both radiometric and photometric. R.H.K. is also responsible for linear polarization calibration observations of "polarization standard" and "null standard" stars.

Richard J. Mitchell was and is responsible for carrying out periodic electrical calibrations. R.J.M. is also responsible for computer interfacing, developing software for observational data reduction and stepping motor drives for both axes of the 72cm aperture Cassegrain Reflector.

Bruce D. Holenstein was responsible for the development of a technique for measurement of the circular polarization simultaneously with that of the linear. B.D.H. was also responsible for the development of a circular polarization calibration technique.

Nicholas M. Elias was responsible for an analysis of the random errors of linear polarization measures and recommendation of the optimum method for data reduction.

10. Acknowledgements

Mrs. Sybil Csigi, Office Administration Assistant, is responsible for the production of this manuscript by "desktop publishing" methods.

Mr. William Barrie (dcd) and Mr. George E. Reahm, both excellent instrument makers, were responsible for the construction of the optical/mechanical parts of the PEMP system.

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